Ideas About Time Contours: From Quantum Chaos to Entropy Inflow

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Based on papers and work in progress with
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Outline

(1) Out-of-time-ordered correlators

(2) k-OTO correlators: fine-grained chaos in AdS_2

(3) 1-OTO correlators: unitarity and KMS condition

(4) Effective field theory: hydrodynamics and entropy inflow

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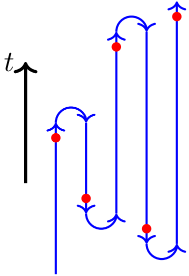
Motivation

- Usually QFT focuses on time-ordered (Feynman) path integrals
- QFT has a lot more correlation functions than the time-ordered ones:

$$\langle \widehat{\mathbb{O}}_1(t_1) \cdots \widehat{\mathbb{O}}_n(t_n) \rangle$$

$\longrightarrow$ $n!$ time orderings



- Seem to be very relevant for **black holes**, many body physics,...
  - **Dissipation, chaos, scrambling,...**
  - Generalized fluctuation relations
  - Usually about QI-theoretic ideas (entanglement, complexity, circuits...)

Schwinger, Keldysh; Feynman–Vernon, '60s

(Maldacena–)Shenker–Stanford '15

Roberts–Yoshida '16, Sekino–Susskind '08

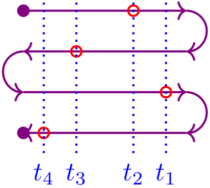
Yunger Halpern '17,...

- Just beginning to understand the relevant physics

OTO contours

- Convenient way to represent n-point function with generic time ordering is the **k-OTO contour**

$$\langle \widehat{\mathbb{O}}_4(t_4)\widehat{\mathbb{O}}_1(t_1)\widehat{\mathbb{O}}_3(t_3)\widehat{\mathbb{O}}_2(t_2)\rangle \;=$$



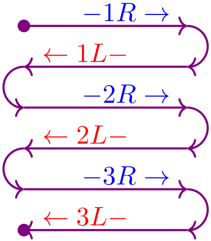
  - Feynman (time-ordered) correlators: 

  - ‘Schwinger-Keldysh’ contour ($k = 1$): 

OTO contours

- Effectively have $2k$ copies of every operator:

$$\widehat{\mathbb{O}} \;\longrightarrow\; \mathbb{O}_{1R}\,,\ \mathbb{O}_{1L}\,,\ \mathbb{O}_{2R}\,,\ \mathbb{O}_{2L}\,,\ \ldots\,,\ \mathbb{O}_{kR}\,,\ \mathbb{O}_{kL}$$

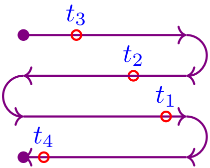


$$\mathcal{Z}_{k-OTO}[\mathcal{J}_{\alpha R}, \mathcal{J}_{\alpha L}] = \text{Tr}\left\{\cdots U^\dagger[\mathcal{J}_{1L}]U[\mathcal{J}_{1R}]\,\rho\, U^\dagger[\mathcal{J}_{kL}]U[\mathcal{J}_{kR}]\cdots\right\}$$

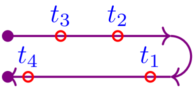
  - Number of n-point functions from k-OTO contour: $(2k)^n$
  - $k = \lfloor \frac{n+1}{2} \rfloor$ allows representation of all $n!$ n-point functions

Redundancies

- $\mathcal{Z}_{k-OTO}[\mathcal{J}_{\alpha R}, \mathcal{J}_{\alpha L}] = \mathrm{Tr}\left\{\cdots U^\dagger[\mathcal{J}_{1L}]U[\mathcal{J}_{1R}]\,\rho\,U^\dagger[\mathcal{J}_{kL}]U[\mathcal{J}_{kR}]\cdots\right\}$
  - Not every n-point function generated from $\mathcal{Z}_k$ is actually k-OTO:



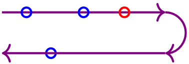
$= \langle 4123 \rangle =$



  - Here: *proper* OTO number of $\langle 4123 \rangle$ is $q = 1$
- Such relations eventually reduce $(2k)^n \longrightarrow n!$ $\qquad (k \sim \lfloor \frac{n+1}{2} \rfloor)$

Largest-time equation

- Refer to these relations as **largest-time equations:** *(c.f., 't Hooft–Veltman '74)*



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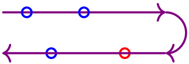
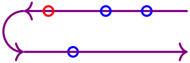


$$\Rightarrow \qquad \langle \mathcal{T}_k\, \mathbb{O}_{\alpha_1}(t_1)\cdots(\mathbb{O}_{pR}-\mathbb{O}_{pL})\,(t_i)\cdots\mathbb{O}_{\alpha_n}(t_n)\rangle = 0$$

if t_i is largest time

- Similarly: **smallest time equations**



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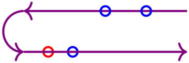


Some counting

- Can do this analysis more carefully and figure out $g_{n,q}$ and $h^{(q)}_{n,k}$ in

$$n! = \sum_{q=1}^{\lfloor \frac{n+1}{2} \rfloor} g_{n,q}\,, \qquad (2k)^n = \sum_{q=1}^{\lfloor \frac{n+1}{2} \rfloor} g_{n,q}\, h^{(q)}_{n,k}$$

  - $n!$ = number of n-point functions
  - $g_{n,q}$ = number of proper q-OTO n-point functions
  - $(2k)^n$ = number of n-point functions naively generated by $\mathcal{Z}_{k-OTO}$
  - $h^{(q)}_{n,k}$ = number of ways to represent <u>any</u> q-OTO n-point function on a k-OTO contour

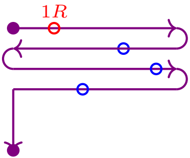
FH–Loganayagam–Narayan–Rangamani '17

Thermal states

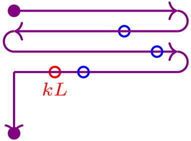
Thermal states: KMS condition

- Take $\rho_{\text{initial}} = \rho_\beta \equiv e^{-\beta \mathbb{H}}$. KMS condition:

$$\text{Tr}\left(\rho_\beta\, \widehat{\mathbb{O}}_1(t_1)\cdots\widehat{\mathbb{O}}_{n-1}(t_{n-1})\widehat{\mathbb{O}}_n(t_n)\right) = \text{Tr}\left(\rho_\beta\, \widehat{\mathbb{O}}_n(t_n - i\beta)\widehat{\mathbb{O}}_1(t_1)\cdots\widehat{\mathbb{O}}_{n-1}(t_{n-1})\right)$$



$= \; e^{-i\beta \frac{d}{dt_n}}$



  - Relates
$\langle \mathbb{O}_1 \cdots \mathbb{O}_n\rangle_\beta \sim \langle \mathbb{O}_n \mathbb{O}_1 \cdots \mathbb{O}_{n-1}\rangle_\beta \sim \langle \mathbb{O}_{n-1}\mathbb{O}_n \mathbb{O}_1 \cdots \mathbb{O}_{n-2}\rangle_\beta \sim \ldots$

  - These relations are k-**OTO** n-**point fluctuation-dissipation relations**, generalizing the well-known statement for $n = 2$:

$$\langle \{\mathbb{A}(t),\, \mathbb{B}(0)\}\rangle = \coth\left(-\frac{i\beta\partial_t}{2}\right)\langle [\mathbb{A}(t),\, \mathbb{B}(0)]\rangle$$

  - For n-point correlators: $n! - (n-1)!$ such relations with *"more (anti-)commutators and more* $\coth$*'s"* *FH–Loganayagam–Narayan–Nizami–Rangamani '17*

Questions

- Is this just combinatorics, or is there physics?

Q: What physics does a "proper q-OTO n-point function" describe?

- The path integral representation $\mathcal{Z}_k$ has a **large redundancy!**
  - But it's useful (e.g., for setting up **effective field theory**)

Q: What's an efficient way to describe the redundancies (beyond counting them)?

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Quantum chaos: 2-OTO 4-point function

- 2-OTO 4-point function:

Kitaev '14

(Maldacena–)Shenker–Stanford '13-'15

$$\frac{\langle W(t)Z(0)W(t)Z(0)\rangle_\beta}{\langle WW\rangle_\beta \langle ZZ\rangle_\beta} \sim 1 - \#\, e^{\lambda_L(t-t_*)}$$

  - Overlap between $|W(t)Z(0)\rangle$ and $|Z(0)W(t)\rangle$
  - λ_L quantifies **quantum chaos** (scrambling time $t_* \sim \frac{\beta}{2\pi}\log N$)
  - Chaos bound: $\lambda_L \leq \frac{2\pi}{\beta}$
  - Time-ordered correlators would not behave like this, e.g.,

$$\frac{\langle W(t)W(t)Z(0)Z(0)\rangle_\beta}{\langle WW\rangle_\beta \langle ZZ\rangle_\beta} \sim 1 \quad \text{for} \quad t \gg \frac{\beta}{2\pi}$$

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- Can extract the connected piece using a commutator:

$$\frac{\langle W(t)[Z(0),W(t)]Z(0)\rangle_\beta}{\langle WW\rangle_\beta \langle ZZ\rangle_\beta} \sim e^{\lambda_L (t-t_*)}$$

The Schwarzian theory

Q: Do higher-point OTOCs contain any more information about chaos?

- To answer this, we *[FH–Rozali '17]* studied the **"Schwarzian theory"**:

$$S = -\frac{1}{\kappa^2}\int du \left[-\frac{1}{2}\left(\frac{t''(u)}{t'(u)}\right)^2 + \left(\frac{t''(u)}{t'(u)}\right)'\right] + S_{matter}$$

- The *time reparameterization mode* $t(u)$ describes:
  - AdS_2 dilaton gravity *[Maldacena–Stanford–Yang '16, Jensen '16, Engelsoy–Mertens–Verlinde '16, ...]*
  - Low energy dynamics of the SYK model *[Maldacena–Stanford '16, ...]*
- This theory is **maximally chaotic**, i.e., $\lambda_L = \frac{2\pi}{\beta}$.

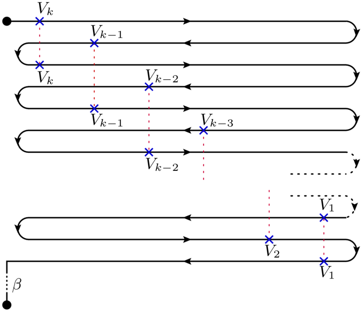
Application: k-OTO scrambling

[FH–Rozali '17]

- Result: there exist **proper k-OTO $2k$-point functions** with...
  - ... exponential growth until $t_*^{(k)} \sim (k-1)t_*$
  - ... Lyapunov exponent same as for 4-point function: $\lambda_L^{(k)} = \lambda_L = \frac{2\pi}{\beta}$

$$\frac{\langle V_1[V_2,V_1][V_3,V_2][V_4,V_3]\cdots[V_k,V_{k-1}]V_k\rangle_{\text{conn.}}}{\langle V_1V_1\rangle\cdots\langle V_kV_k\rangle} \sim \mathcal{O}(1)\times e^{\lambda_L[(t_1-t_k)-(k-1)t_*]}$$

- These correlators start off even smaller (at $\mathcal{O}(\frac{1}{N^{k-1}})$), so they can **grow for longer**.
- Interpretation: measure the scrambling of increasingly fine-grained quantum information *c.f., Roberts–Yoshida '16, Cotler–Hunter-Jones–Liu–Yoshida '17,...*



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Consider 1-OTO path integrals
(aka Schwinger-Keldysh formalism)

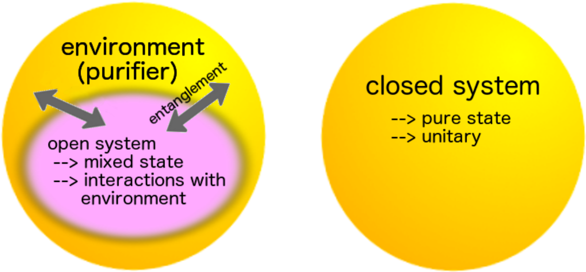
There might well be k-OTO generalizations of many statements.

A different perspective

- **Mixed state** can be purified by doubling Hilbert space: $\mathcal{H} \ \rightarrow \ \mathcal{H}_L \otimes \mathcal{H}_R$
- Schwinger-Keldysh path integral describes **evolution of mixed state** ρ:

$$\mathcal{Z}_{SK}[\mathcal{J}_R, \mathcal{J}_L] = \mathsf{Tr}\left\{U[J_R]\,\rho\, U^\dagger[J_L]\right\}$$

- Real time dynamics of mixed states is very rich: **open systems**



  - **Lorentzian** dynamics (possibly far from equilibrium)
  - **Dissipation, entropy, "information loss",...**

Entropy

- Coarse-grained entropy:
  - Low energy fluctuations dissipate (UV/IR correlations)
  - Subject to $\Delta S \geq 0$
  - Emergent entropic arrow of time: ‘looks’ non-unitary

- Underlying microscopic QFT is unitary
- Clash of **low-energy effective field theory** and **unitarity**

Q: Can we use formal insights from OTO path integrals to learn something about the **emergence of entropy and dissipation**?

- I'll now try to motivate that redundancies in the Schwinger-Keldysh path integral description can be encoded by:
  - Increasing the field content even further, and
  - Imposing an $\mathcal{N}_T = 2$ superalgebra symmetry structure.
- Implementing this in an effective field theory of thermal states (hydrodynamics) will lead to a **Wilsonian QFT picture of coarse grained entropy.**

A consequence of unitarity

- The redundancies in OTO contour representation are really a manifestation of **unitarity** ($U[\mathcal{J}]U^\dagger[\mathcal{J}] = 1$):

$$\mathbb{Z}_1[\mathcal{J}_R, \mathcal{J}_L] = \mathsf{Tr}\left\{U[\mathcal{J}_R]\, \rho\, U^\dagger[\mathcal{J}_L]\right\}$$

$$\mathbb{Z}_1[\mathcal{J}_R = \mathcal{J}_L \equiv \mathcal{J}] = \mathsf{Tr}\,(\rho) \qquad (\text{“localization”})$$

The latter is still a **theory of difference operators:**

$$\int \mathcal{J}_R\, \mathbb{O}_R - \mathcal{J}_L\, \mathbb{O}_L \xrightarrow{\mathcal{J}_R = \mathcal{J}_L \equiv \mathcal{J}} \int \mathcal{J}(\mathbb{O}_R - \mathbb{O}_L)$$

$\Rightarrow$ vanishing of **difference operator correlators**:

$$\langle \mathcal{T}_{\mathcal{C}}\, \mathbb{O}^{(1)}_{\mathsf{diff}} \cdots \mathbb{O}^{(n)}_{\mathsf{diff}} \rangle = 0 \qquad (\mathbb{O}_{\mathsf{diff}} \equiv \mathbb{O}_R - \mathbb{O}_L) \qquad (*)$$

Unitarity and cohomology

$$\langle \mathcal{T}_{\mathcal{C}} \; \mathbb{O}^{(1)}_{\text{diff}} \cdots \mathbb{O}^{(n)}_{\text{diff}} \rangle = 0 \qquad (\mathbb{O}_{\text{diff}} \equiv \mathbb{O}_R - \mathbb{O}_L) \qquad (*)$$

- Proposal *[FH–Loganayagam–Rangamani '15]*:
  $(*)$ happens because $\mathbb{O}_{\text{diff}}$ is trivial element of a BRST cohomology
  - I.e., we want to write: $\mathbb{O}_{\text{diff}} = Q_{\text{BRST}}(\mathbb{O}_{\overline{G}}) = \overline{Q}_{\text{BRST}}(\mathbb{O}_G)$
  - $(*)$ would then be a (topological) symmetry statement
  - $\mathbb{O}_G, \mathbb{O}_{\overline{G}}$ will have to be **ghost/anti-ghost**

- Compare with topological QFT (or gauge theory):
  - characterize topological operators (or pure gauge states) using nilpotent, Grassmann-odd BRST charges: $Q_{\text{BRST}}(\ldots)$, $\overline{Q}_{\text{BRST}}(\ldots)$
  - BRST cohomology characterizes physical states
  - **correlators of BRST-exact fields vanish**

Universal Schwinger-Keldysh supergeometry

- SK cohomology: lift every operator $\mathbb{O}$ to a **BRST multiplet** $\{\mathbb{O}_R, \mathbb{O}_L, \mathbb{O}_G, \mathbb{O}_{\bar{G}}\}$ with $(\mathbb{O}_R - \mathbb{O}_L) = \mathcal{Q}_{SK}(\mathbb{O}_{\bar{G}}) = \overline{\mathcal{Q}}_{SK}(\mathbb{O}_G)$
  - This is in a sense a **covariant** SK formalism
  - Usual SK theory is a gauge fixed version ($\mathbb{O}_G = \mathbb{O}_{\bar{G}} = 0$)

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- Trick: quadrupling of operator algebra $\Leftrightarrow$ **operator superalgebra**

$$\mathring{\mathbb{O}} = \frac{\mathbb{O}_R + \mathbb{O}_L}{2} + \theta\, \mathbb{O}_{\overline{G}} + \bar{\theta}\, \mathbb{O}_G + \bar{\theta}\theta\, (\mathbb{O}_R - \mathbb{O}_L) \qquad \theta^2 = \bar{\theta}^2 = 0$$

$\overline{\mathcal{Q}}_{SK}$ $\quad$ $\mathcal{Q}_{SK}$ $\quad$ $\mathcal{Q}_{SK}$ $\quad$ $\overline{\mathcal{Q}}_{SK}$

  - $\mathcal{Q}_{SK} \sim \partial_{\bar{\theta}}$ and $\overline{\mathcal{Q}}_{SK} \sim \partial_\theta$
  - **Super-translation invariance** $\leftrightarrow$ BRST symmetry $\leftrightarrow$ unitarity

unitarity of SK theory $\Rightarrow$ correlators of BRST-exact operators vanish

FH–Loganayagam–Rangamani '16, Geracie–FH–Loganayagam–Narayan–Ramirez–Rangamani '17

Features of Schwinger-Keldysh QFT

Unitarity leads to constraints in SK path integral

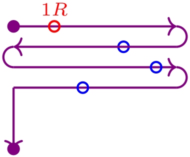
- Respecting these constraints is equivalent to imposing a certain BRST symmetry $\{\mathcal{Q}_{SK}, \overline{\mathcal{Q}}_{SK}\}$
- Implement by lifting to operator super-algebra and work in superspace: $\mathcal{Q}_{SK} \sim \partial_{\bar{\theta}}\,,\ \overline{\mathcal{Q}}_{SK} \sim \partial_{\theta}$

Thermal states

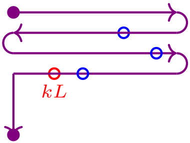
- Consequence of KMS condition:

$$\langle \mathcal{T}_{\mathcal{C}}\ \widetilde{\mathbb{O}}_{\mathsf{diff}}(t_1)\cdots\widetilde{\mathbb{O}}_{\mathsf{diff}}(t_n)\rangle = 0$$

where $\widetilde{\mathbb{O}}_{\mathsf{diff}}(t) \equiv \mathbb{O}_R(t) - \mathbb{O}_L(t-i\beta)$



$$= \ e^{-i\beta\frac{d}{dt_n}}$$



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$$\langle \mathcal{T}_{\mathcal{C}}\, \widetilde{\mathbb{O}}_{\text{diff}}(t_1) \cdots \widetilde{\mathbb{O}}_{\text{diff}}(t_n) \rangle = 0$$

where $\widetilde{\mathbb{O}}_{\text{diff}}(t) \equiv \mathbb{O}_R(t) - \mathbb{O}_L(t - i\beta)$

  - Can again encode this by setting $\widetilde{\mathbb{O}}_{\text{diff}} = \mathcal{Q}_{_{KMS}}(\ldots) = \overline{\mathcal{Q}}_{_{KMS}}(\ldots)$ on the same field content as before
  - $\{\mathcal{Q}_{_{SK}}, \overline{\mathcal{Q}}_{_{SK}}, \mathcal{Q}_{_{KMS}}, \overline{\mathcal{Q}}_{_{KMS}}\}$ generate $\mathcal{N}_T = 2$ **superalgebra**

$$\{\mathcal{Q}_{_{KMS}}, \overline{\mathcal{Q}}_{_{SK}}\} = \{\overline{\mathcal{Q}}_{_{KMS}}, \mathcal{Q}_{_{SK}}\} = \Delta_\beta$$

Vafa–Witten '94

Dijkgraaf–Moore '97

FH–Loganayagam–Rangamani '15

    - ⋆ All other commutators vanish
    - ⋆ $\Delta_\beta \equiv (1 - e^{-i\beta\partial_t})$ roughly measures thermal fluctuations
  - Formally: as if Δ_β was generator of (gauge) group action
    - ⋆ "Equivariant cohomology": would tell us how to be covariant w.r.t. Δ_β
    - ⋆ At high temperatures ($\Delta_\beta \approx i\,\beta\,\partial_t$): algebra would describe a **gauge theory of thermal time translations**

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- Implement by lifting to operator super-algebra and work in superspace: $\mathcal{Q}_{_{SK}} \sim \partial_{\bar{\theta}}\,,\ \overline{\mathcal{Q}}_{_{SK}} \sim \partial_{\theta}$

KMS condition leads to further constraints

- Can be imposed by "covariantizing" the BRST structure w.r.t. operators such as $\Delta_\beta = 1 - e^{-i\beta\partial_t} \approx i\beta\partial_t$

$\Rightarrow \mathcal{N}_T = 2$ symmetry algebra of thermal translations

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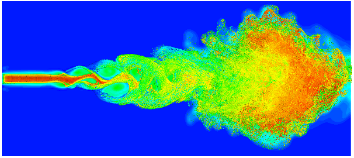
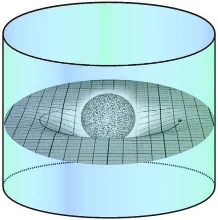
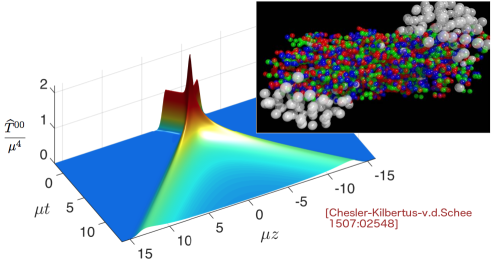
Hydrodynamics

- Hydrodynamics: effective field theory of low energy dynamics
  - long wavelength fluctuations over a thermal state
  - universal framework for wide variety of systems
  - old subject, phenomenology well understood







- **Holography:** strongly coupled hydrodynamics of CFT$_d$ is *dual* to gravitational physics of large AdS$_{d+1}$ black holes

Proposal for EFT in local equilibrium

(1) Take system in **local equilibrium** with effective degrees of freedom Φ

(2) Introduce superspace $x^I = (x^a, \theta, \bar{\theta})$

(3) Lift fields to superfields:

$$\mathring{\Phi} = \frac{\Phi_R + \Phi_L}{2} + \theta\,[\Phi_{\bar{G}} + \ldots] + \bar{\theta}\,[\Phi_G + \ldots] + \bar{\theta}\theta\,[(\Phi_R - \Phi_L) + \ldots]$$

(4) Impose $\mathcal{N}_T = 2$ supersymmetry with gauge group generator:

$$\Delta_\beta = 1 - e^{-i\beta\partial_t} \longrightarrow i\beta\partial_t \qquad (\beta\omega \gg 1)$$

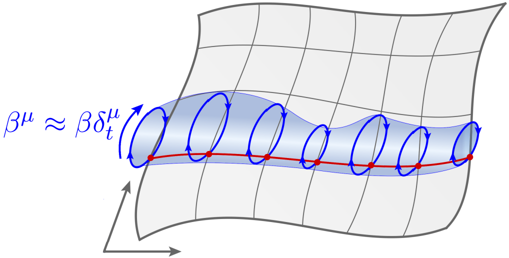
$\Rightarrow$ gauge (super-)field $\mathring{\mathcal{A}}_I$ and covariant derivatives $\mathring{\mathcal{D}}_I = \partial_I + (\mathring{\mathcal{A}}_I, \cdot\,)$

(5) Write most general effective actions with these fields and symmetries:

$$S_{SK} = \int d^d x\, d\theta\, d\bar{\theta}\; \mathcal{L}(\mathring{\Phi}, \mathring{\mathcal{A}}_I, \mathring{\mathcal{D}}_I)$$

(6) De-align sources $J_{\substack{R\\L}} \to J \pm \frac{1}{2}\,\tilde{J}$

Spacetime picture with $U(1)_T$



- Gauge 'thermal diffeomorphisms' $\pounds_\beta \longrightarrow U(1)_T$ symmetry
  - Like *local* Euclidean periodicity
- Spacetime carries 1-diml. $U(1)_T$ fibres ($\sim$ thermal direction)
  - Like a *local* Euclidean thermal circle
- Spatial slice: a section $\mathcal{M}/U(1)_T$
  - Like *local* Kaluza-Klein reduction

Effective field theory of hydrodynamics

- Hydrodynamics is a theory of currents:

$$T^{ab}[T(x), u^a(x), g_{ab}(x)] = \varepsilon\, u^a u^b + p\,(g^{ab} + u^a u^b) + T^{ab}_{(1)} + T^{ab}_{(2)} + \ldots$$

$$J^a_S[T(x), u^a(x), g_{ab}(x)] = s\, u^a + J^a_{S,(1)} + J^a_{S,(2)} + \ldots$$

  - Infinite number of terms, all explicitly constructed and classified

    FH–Loganayagam–Rangamani '15

  - Second law $\nabla_a J^a_S \geq 0$ gives very non-trivial constraints (e.g. $\eta, \zeta \geq 0$)

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Theorem *[FH–Loganayagam–Rangamani, '15 & w.i.p.]*

The most general hydro effective action with $\mathcal{N}_T = 2$ symmetry of thermal translations (+ diff.invariance + CPT) describes:

- All hydrodynamic $\{T^{ab}, J^a_S\}$ at all orders in derivative expansion, which are consistent with 2nd law
- No $\{T^{ab}, J^a_S\}$ inconsistent with second law
- Fluctuations (Schwinger-Keldysh difference fields)
- ...

see also: *Crossley-Glorioso-Liu '15, Jensen-Pinzani–Fokeeva–Yarom '17*

Entropy inflow

FH–Loganayagam–Rangamani '18

Jensen–Marjieh–Pinzani-Fokeeva–Yarom '18

- The $U(1)_T$ symmetry Noether current is the **superspace free energy current** $N^I = J_S^I + T^{IJ}\beta_J \quad (z^I = (\sigma^a, \theta, \bar{\theta}))$
  - Familiar from stationary black holes: Wald entropy is a Noether charge

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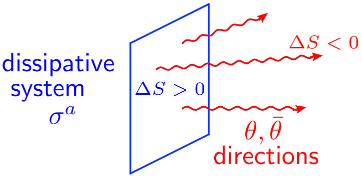
  - Being a symmetry current in superspace, it is super-conserved:

$$0 = \mathcal{D}_I N^I \equiv \underbrace{\mathcal{D}_a N^a}_{\substack{\geq 0 \\ \text{(2nd law)}}} + \underbrace{\mathcal{D}_\theta N^\theta + \mathcal{D}_{\bar{\theta}} N^{\bar{\theta}}}_{\leq 0} + \text{SK ghosts}$$

  - **Inflow mechanism for entropy** "restores" unitarity:



total: $(\Delta S)_{\{x^\mu\}} + (\Delta S)_{\{\theta,\bar{\theta}\}} = 0$

(upto ghosts and fluctuations)

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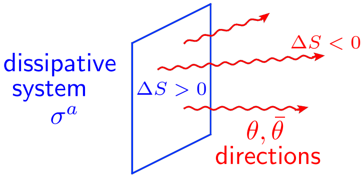

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(upto ghosts and fluctuations)

- Bonus: $\mathfrak{F}_{\theta\bar{\theta}}$ multiplies everything dissipative and should be CPT invariant. A CPT breaking $\langle \mathfrak{F}_{\theta\bar{\theta}} \rangle = -i$ serves as an **order parameter for dissipation**

Real time EFT: summary

- **Step 1:** Identified **symmetry principles** in OTO path integrals, which encode unitarity and KMS condition
- **Step 2:** Proposed a way to implement these symmetries in near-equilibrium EFT
  - **Cohomological (susy) structure** with $U(1)_T$ gauge theory of entropy
- **Step 3:** Check this proposal in examples
  - Brownian motion: detailed completion of a well-known story
  - Hydrodynamics: will provide very non-trivial check (reproduce complete solution and classification of hydrodynamic transport)
  - What about gravity?

Summary

- OTOCs provide rich set of QFT observables.
- We identified a particular set of k-OTO $2k$-point functions, which can be computed in the Schwarzian theory. They provide **a hierarchy of increasingly fine grained measures of quantum scrambling.**
- Constraints on the SK path integral from unitarity can be phrased as a **topological BRST symmetry.**
- With further constraints from the KMS condition, find an $\mathcal{N}_T = 2$ **symmetry structure of thermal translations.**
- In the long-wavelength limit such a symmetry principle constrains **dissipative effective actions** in precisely the right way
  - Reproduce classification of hydrodynamics
  - **Inflow mechanism** for hydrodynamic entropy

Further Details

Toy model: Langevin dynamics

- Consider Brownian motion of particle at $x(t)$ in viscous medium:

$$-\text{Eom} \equiv m\,\frac{d^2x}{dt^2} + \frac{\partial U}{\partial x} + \nu\,\frac{dx}{dt} = \mathbb{N}$$

- Martin-Siggia-Rose (MSR) construction:

$$\begin{aligned}
&[dx]\int[d\mathbb{N}]\,\delta(\text{Eom}+\mathbb{N})\,\text{det}\left(\frac{\delta\text{Eom}}{\delta x}\right)\,e^{i\,S_{\text{Gaussian noise}}[\mathbb{N}]} \\
&=[dx]\int[df][d\overline{\psi}][d\psi]\ \text{exp}\ i\int dt\left(f\,\text{Eom}+i\,\nu\,f^2+\overline{\psi}\left(\frac{\delta\text{Eom}}{\delta x}\right)\psi\right)
\end{aligned}$$

- Can write this in terms of $\mathcal{N}_T = 2$ supercharges $\overline{\mathcal{Q}}$, $\mathcal{Q}$, implementing the algebras of before:

$$=[dx]\int[df][d\overline{\psi}][d\psi]\ \text{exp}\ i\int dt\left\{\overline{\mathcal{Q}}\,,\left[\mathcal{Q}\,,\frac{m}{2}\,\left(\frac{dx}{dt}\right)^2-U(x)-i\,\nu\,\overline{\psi}\psi\right]\right\}\Bigg|_{\substack{\text{gauge}\\\text{fixed}}}$$

Toy model: Langevin dynamics

- Can make susy manifest by working in superspace:

$$\overline{\mathcal{Q}} = \partial_\theta$$

$$\mathring{x} \;=\; x \;+\; \theta\,\overline{\psi} \;+\; \bar{\theta}\,\psi \;+\; \theta\bar{\theta}\,f$$

$$\mathcal{Q} = \partial_{\bar{\theta}}$$

$$\int dt \left\{ \overline{\mathcal{Q}} \,, \left[\mathcal{Q} \,, \frac{m}{2} \left(\frac{dx}{dt} \right)^2 - U(x) - i\,\nu\,\overline{\psi}\psi \right] \right\} \Bigg|_{\substack{\text{gauge}\\ \text{fixed}}}$$

$$= \int dt\, d\theta\, d\bar{\theta} \left(\frac{m}{2} \left(\frac{d\mathring{x}}{dt} \right)^2 - U(\mathring{x}) - i\,\nu\,\mathcal{D}_\theta \mathring{x}\; \mathcal{D}_{\bar{\theta}} \mathring{x} \right) \Bigg|_{\substack{\text{gauge}\\ \text{fixed}}}$$

  - Taking the superspace and $U(1)_T$ seriously, we could have guessed this action immediately!
  - **Ghost bilinear** term responsible for dissipation

Toy model: Langevin dynamics

- Equation of motion for x from this action:

$$m\frac{d^2x}{dt^2} + \frac{\partial U(x)}{\partial x} + i\,\langle \mathcal{F}_{\bar\theta\theta}\rangle\,\nu\,\frac{dx}{dt} = 2\,i\,\nu\,f$$

  - $\langle \mathcal{F}_{\bar\theta\theta}\rangle = -i$: **order parameter for dissipation**
  - This **spontaneously breaks CPT**
  - Supersymmetry Ward identity $\Leftrightarrow$ **Jarzynski relation:**

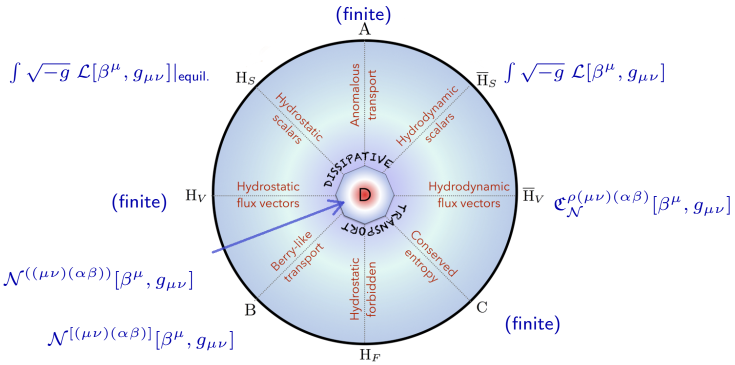
$$\left\langle e^{-\beta\,\Delta W}\right\rangle = e^{-\beta\,\Delta F} \quad \Rightarrow \quad \langle \Delta W\rangle \geq \Delta F$$

Jarzynski '97

Crooks '98

    - $\Delta F =$ difference of free energies between initial and final state
    - $\langle \cdots \rangle$: statistical average for going from state $A \to B$
    - $\Delta W = \int_{t_A}^{t_B} dt\,\frac{\partial U(x(t),t)}{\partial t} =$ total work done

Eightfold classification of hydrodynamic transport



FH-Loganayagam-Rangamani '14 '15

Theorem: The eightfold way of hydrodynamic transport

- There are eight classes of $\{T^{\mu\nu}, J_S^\mu\}$ consistent with $\nabla_\mu J_S^\mu \gtrsim 0$.
- All of them can be constructed easily at all orders in ∇_μ.
- Constitutive relations not produced by this algorithm, are forbidden by second law (Class H_F).

Generalized fluctuation relations

- Fluctuation-dissipation theorem for 2-point functions:

$$\langle\{\mathbb{O}_1(t_1), \mathbb{O}_2(t_2)\}\rangle_\beta = \coth\left(-\frac{i\beta}{2}\frac{d}{dt_2}\right)\langle[\mathbb{O}_1(t_1), \mathbb{O}_2(t_2)]\rangle_\beta$$

- KMS condition gives FD theorems for n-point functions
  - Express $n!$ Wightman correlators in terms of $(n-1)!$ spectral functions
  - Theorem: can choose as independent spectral functions the following:

$$\langle\left[\cdots\left[\left[\mathbb{O}_n,\, \mathbb{O}_{\pi(1)}\right],\, \mathbb{O}_{\pi(2)}\right],\cdots,\mathbb{O}_{\pi(n-1)}\right]\rangle_\beta$$

for $\pi \in S_{n-1}$